A NUMERICAL SOLUTION OF THE PROBLEM OF LAMINAR JET DEVELOPMENT IN A CHANNEL

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The results are given of a computer calculation of the Navier-Stokes equations for the flow of a viscous incompressible conducting fluid, arising when a flat laminar jet flows into a channel of finite width under the influence of a magnetic field (for $R_m \ll 1$).

The problem reduces to the following. A jet of fluid flows into a flat channel with a transverse dimension of unity, through a slit whose width is one-tenth of the channel width. An external uniform magnetic field (the case of small values of the magnetic Reynolds' number $R_m \ll 1$) is in the direction transverse to the channel. The induced currents are assumed to be short circuited through electrodes (the side walls of the channel). The initial velocity profile is taken to be uniform (the value of the velocity at the entrance is equal to unity). In the calculations the position of the slit on the end wall of the channel is varied relative to its axis, and the values of the Reynolds' number R and Hartmann number H are also varied.

Concrete calculations (for R = 50) carried out for various values of the dimensionless distance of the slit axis from the channel axis ($\bar{y}$ = 0, 0.1, 0.2, 0.3, 0.4, 0.45) and several values of the Reynolds' and Hartmann numbers.

The initial equations for nonsteady flow are

$$\frac{\partial u}{\partial t}+R\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-Hu \qquad \left(H=\frac{\sigma B^2L^2}{\eta}\right),$$

$$\frac{\partial v}{\partial t}+R\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial y^2}, \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

Introducing the stream function $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$ we obtain

$$\frac{\partial\varphi}{\partial t}+R\left(\frac{\partial\psi}{\partial y}\frac{\partial\varphi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}\right)=\nabla^2\varphi-H\frac{\partial^2\psi}{\partial y_2}, \qquad \Delta^2\psi=\varphi. \tag{1}$$

The equations are written in dimensionless form; the velocity and dimension scales are, respectively, the velocity at the exit from the slit and the width of the slit under consideration.

In the finite-difference representation Eqs. (1) have the form

$$\begin{aligned}\varphi_{i,k}^{n+1}=\Bigg[\varphi_{i,k}^{n}+\Delta t\Bigg\{&\frac{\varphi_{i+1,k}^{n}+\varphi_{i-1,k}^{n}}{(\Delta x)^2}+\frac{[(\varphi_{i,k+1}^{n}+\varphi_{i,k-1}^{n})+H(\psi_{i,k+1}^{n}+\psi_{i,k-1}^{n})](\Delta x)^2}{(\Delta x)^2(\Delta y)^2}-\\ &-2\frac{[(\Delta x)^2+(\Delta y)^2]\varphi_{i,k}^{n}+H\psi_{i,k}^{n}(\Delta x)^2}{(\Delta x)^2(\Delta y)^2}-R\left(\frac{\psi_{i,k+1}^{n}-\psi_{i,k-1}^{n}}{2\Delta y}\,\frac{\varphi_{i+1,k}^{n}-\varphi_{i-1,k}^{n}}{2\Delta x}-\right.\\ &\left.-\frac{\psi_{i+1,k}^{n}-\psi_{i-1,k}^{n}}{2\Delta x}\,\frac{\varphi_{i,k+1}^{n}-\varphi_{i,k-1}^{n}}{2\Delta y}\right)\Bigg\}\Bigg],\end{aligned} \tag{2}$$

$$\psi_{i,k}^{n+1}=\frac{(\psi_{i+1,k}^{n+1}+\psi_{i-1,k}^{n+1})(\Delta y)^2+(\psi_{i,k+1}^{n+1}+\psi_{i,k-1}^{n+1})(\Delta x)^2}{2[(\Delta x)^2+(\Delta y)^2]}-\frac{(\Delta x)^2(\Delta y)^2\varphi_{i,k}^{n+1}}{2[(\Delta x)^2+(\Delta y)^2]}. \tag{3}$$

The finite-difference equations were solved on an electronic computer by the method employed in the papers of Simuni* [1, 2].

The boundary conditions for the functions ψ and φ in finite-difference representation may be expressed as follows.

The boundary contour consists of the three channel walls (with the slit on the end wall) and the normal plane to

*We wish to thank L. M. Simuni for advice and help in carrying out the present work.

the channel axis downstream. The latter is chosen in the region of steady-state Hartmann flow. As usual in this case the velocity vector is equal to zero for a viscous fluid at the solid walls, while the velocity profile within the limits of the slit is uniform and varies in time according to the law $V = 1 - e^{-kt}$ (k is a constant). Finally, a Hartmann flow velocity profile is assumed for the downstream boundary with a fluid flow rate varying with time. This profile was found graphically from the solution of the finite-difference equations for uniform flow. These conditions specify the distribution of the stream function in time over the whole bounding contour.

The boundary conditions for the function φ, in accordance with [1], are determined from the formula

$$\varphi = 2\,\frac{\psi_w^{(n)} - \psi_i^{(n)}}{(\Delta y)^2}.$$

Here ψ_w and ψ_i are the values of the stream functions at the wall and in the adjacent (removed by one step) layer, respectively.

Briefly the order of the calculations reduces to the following. The value of the function $\varphi_{i,k}^{n+1}$ is found via Eq. (2) from the known values $\psi_{i,k}^{n}$ and $\varphi_{i,k}^{n}$ for the preceding instant. The value $\psi_{i,k}^{n+1}$ is then found by integrating Eq. (3). The calculations were continued until steady state flow was obtained.

As regards choice of step length we note that the calculations showed that a step of $\Delta x = \Delta y = 5 \cdot 10^{-3}$ was sufficient. The time step length Δt and the constant k were chosen so as to ensure that the variation of the functions was sufficiently smooth.

The results of one of the series of calculations is given in Fig. 1a–f for the values R = 50, H = 0, which corresponds to developed flow with circulatory zones in the corners of the channel. The solid lines on the figures represent the streamlines while the velocity profiles at several transverse cross sections are given by the dashed lines.

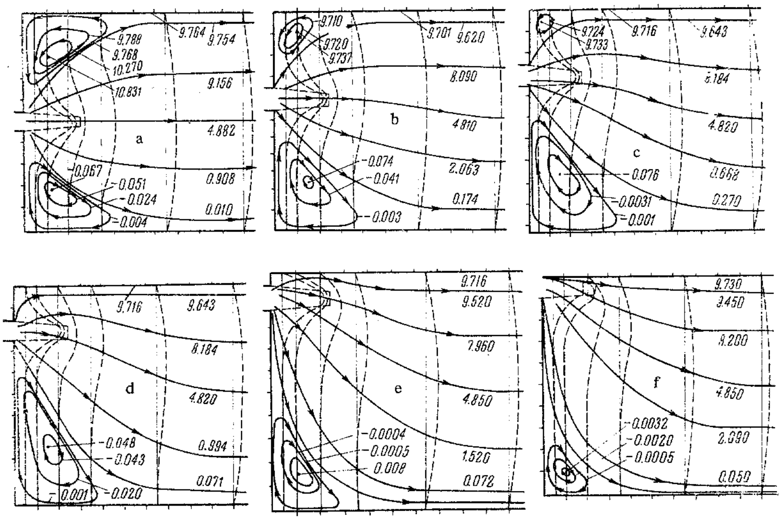

Fig. 1

It is clear from the graphs that as the axis of the slit is displaced from the channel axis (Fig. 1a) in the direction of the upper wall the lower vortex increases noticeably, while the upper vortex suffers a corresponding rapid decrease (in Fig. 1d–f it is absent altogether).

Some data are given in Fig. 2 characterizing the intensity of the circulatory motion in the stagnation zones, in the corners of the channel for various positions of the slit. The solid line shows the relative flow rate of fluid circulating in the stagnation zone (Q_0, the flow rate of fluid flowing into the channel through the slit is taken as the scale). The dashed line shows the variation of the geometrical characteristics of the stagnation zone (the area bounded by the zero or the maximum streamline) measured relative to an arbitrary unit area (a square with a side equal to the channel width).

These curves provide an additional visual illustration of the asymmetry of the flow, which is already clear from Fig. 1.

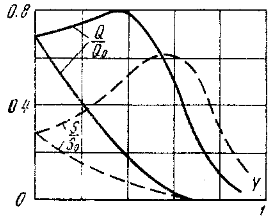

Fig. 2

The flow picture for the values R = 200, H = 0 is given in Fig. 3a for a symmetrical slit position. In this case (compared with flow for R = 50) the relative dimensions of the stagnation zones and the intensity of the circulation in them increase equally. For small values of R (roughly up to R ≈ 20) the stagnation zones are absent, and the flow around the corners is almost smooth.

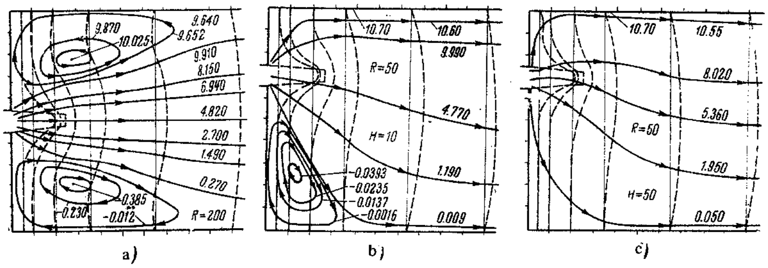

Fig. 3

Let us see how this picture changes under the influence of an applied transverse magnetic field. Applying a transverse magnetic field decreases the dimensions and intensity of the circulatory zones, until they disappear completely. In all cases (symmetric or asymmetric flow) this effect is more strongly marked for larger values of the Hartmann number and smaller values of the Reynolds' number.

As an example Fig. 1c and Fig. 3b, c show the streamlines for the case of asymmetric slit position. The data given in Fig. 1c and Fig. 3b, c, refer to R = 50 and values of H = 0.10 and 50. It is clear that the lesser circulatory zone is damped at first (in the upper corner of the channel in Fig. 3b), and subsequently as the number H increases further the lower zone also disappears.

Thus the transverse field which retards the flow leads to a decrease of reverse flow in the corners of the channel, and eventually to smooth motion without detachment. As opposed to this a longitudinal field increases the circulatory zones but a noticeable effect is observed for considerably larger values of the Hartmann number.

These effects should be preserved qualitatively for turbulent motion. On the whole the data obtained enables us to construct an intuitive kinematic picture of the flow under consideration. Without going into details we note that the formation of stagnation and circulatory zones in the corners of the channel (which remain for turbulent flow) and the possibility of influencing them actively have an important significance for some applications, such as the stabilization of flames, etc.

REFERENCES

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